Lecture 20

Thursday, March 18, 2021 4:07 PM

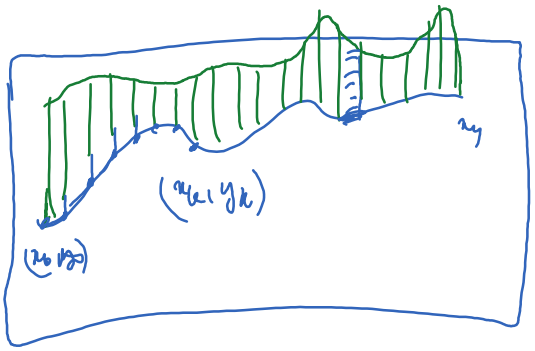
Plot vector fields on Mathematica:

VectorPlot [{x^2, xy}, {x, 0, 1}, {y, 0, 1}]

Line integral

integral
- "single" integral $\int_a^b$, $\int_{[a,b]}$
- double integral $\iint_D$
- triple integral $\iiint_E$
- iterated integral $\int_a^b \int_c^d \cdots$

Can we integrate a function $f(x,y)$ over a curve instead of a region?



$$s(t) = \int_a^t |r'(\tau)|\, d\tau$$

$$ds = s'\,dt = |r'(t)|\,dt$$

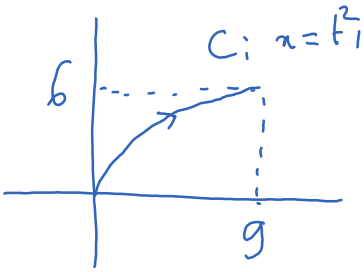
$$\sum f(x_k, y_k)\, |r'(t_k)|\, \Delta t$$

$$\approx \int_a^b f(x(t), y(t))\, \underbrace{|r'(t)|\, dt}_{ds}$$

$$= \text{“} \int_C f(x,y)\, ds \text{”}$$

integral of a scalar function over a curve.

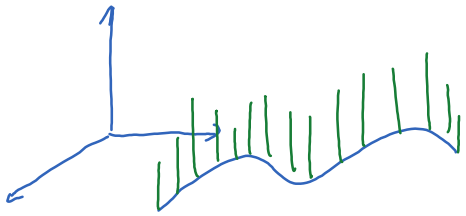
Ex



$$\int_C y\,ds = ?$$

Mathematica:

How to plot function $f(x,y)$ on top of a curve C?



$C: (x(t), y(t), 0)$

Surface: $(x(t), y(t), s)$

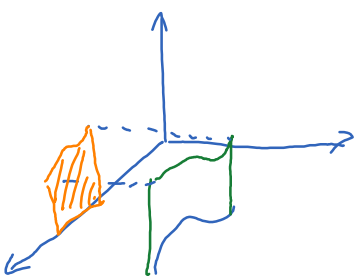
$t \in [a,b]$, $s \in [0, f(x(t), y(t))]$

ParametricPlot3D[{t^2, 2t, s}, {t, 0, 3}, {s, 0, 2t}]

$\int_C f(x,y)\,ds$ is the area of the wall.

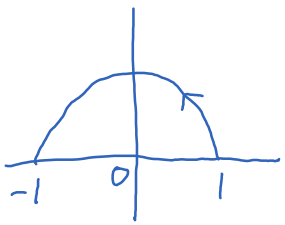
or mass of a wire

* Line integral writ x or y



$\int_C f(x,y)\,dx$ = area of the shadow of the fence.

Ex $f(x,y) = x+y$



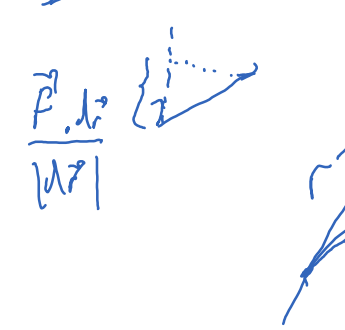
$$\int_C (x+y)\,dx = \int_0^{\pi} (\cos t + \sin t)(-\sin t)\,dt$$

$$= \cdots\cdots$$

Line integral of a vector field

Work along a curve

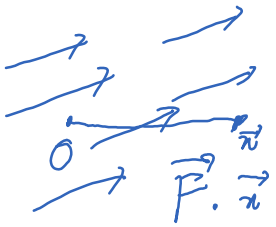
$$\int \vec{F}\cdot d\vec{r} = \int \vec{F}\cdot \frac{d\vec{r}}{|d\vec{r}|}\,|d\vec{r}|$$



$$\int \frac{F\cdot r'}{|r'|}\,ds$$

$$\frac{r'}{|r'|}\,ds = r'\,dt = dr$$

$|r'|\,dt$



Ex $F(x,y,z) = \langle xz, y, 1\rangle$

Along the curve C: $\begin{cases} x = t \\ y = 2t \\ z = t^2 \end{cases}$ $0 \le t \le 1$